Computation of the Stress Intensity Factor in Bonded Repairs to Acoustically Fatigued Panels

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ABSTRACT

The application of bonded repairs to structures subject to acoustic fatigue is investigated, with a view to developing simple solutions suitable for engineering applications. Acoustic fatigue is caused by a random response of a structure due to time varying pressure waves from engine and/or aerodynamic sources. For vibration problems the use of damping in repairs will reduce the amplitude of vibration and hence the stress intensity factor in the cracked structure. To assess the viability of the composite bonded repairs subject to acoustic fatigue new analytical tools are needed. In this paper the use of a simplified dynamic analysis combined with crack-bridging theory allows the influence of damping to be assessed. This theory is adapted to panels with various edge support conditions with good results. Validation of this work is carried out using a Finite Element (F.E.) analysis in which three-dimensional brick elements are used to model the plate, adhesive and boron patch, and use is made of the power spectral density capability of the NASTRAN program.

RELEASE LIMITATION

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Computation of the Stress Intensity Factor in Bonded Repairs to Acoustically Fatigued Panels

Executive Summary

The repair of cracked aircraft structures subject to manoeuvre-type loads using composite bonded repairs has resulted in considerable aircraft life time extension and hence cost savings. However the use of bonded patches to repair panels with acoustically induced cracks (acoustic fatigue) is only recent and presents some difficulties. Acoustic fatigue is a result of high frequency lateral vibration of an aircraft panel as a result of intense random noise caused by engine and/or aerodynamic effects. For example acoustically induced cracks have been found in the lower external surface of the nacelle skin of the F/A-18 aircraft. In this case the overall sound pressure levels of the order of 170 db have been recorded. Attempts to repair these cracks by applying standard designs of composite bonded repair were made, however the cracks continued to grow. These panels were repaired on the basis of high level manoeuvre loads but low frequency out-out-of plane loads. It is evident that the use of composite bonded repairs to cracked panels subject to acoustic excitation requires analytical tools that take high frequency random out-of-plane vibration into account.

Previous work has shown that while the boron fibre composite bonded repairs applied to panels containing acoustically induced cracks do reduce the driving force at the crack (stress intensity), the high number of cycles, due to the high loading frequencies, lead to significant crack growth. For example, one hour of flying may result in about 10^6 cycles for the high frequency acoustic fatigue case. Since it is known that the amplitude of vibration of the panel is dependent on the damping, then increasing the damping may reduce the crack growth rate to an acceptable level.

In this report an analytical solution is proposed to allow the computation of stress intensity in highly damped repairs, which involves the use of crack-bridging theory. This analytical approach is validated against detailed 3D finite element analysis. Furthermore a simplified dynamic analysis of the random loading is adopted. It is shown that a combination of the simplified dynamic analysis and crack bridging theory is applicable to infinitely wide panels. It has been shown in previous work that a simple plate analysis for crack bridging can predict the behaviour for plates in cylindrical bending for the static case. Furthermore use of simple plate bending theory can extend this theory to rectangular panels restrained on all edges. As a result this theory, together with appropriate crack growth data and damping data, will enable the design of highly damped repairs of acoustically-induced cracked panels of aircraft structures.

Authors

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Mr. R.J.Callinan is a senior research scientist and graduated from RMIT (Aero. Eng.) in 1969 and from Monash University in 1971 (Civil. Eng.) and completed a M.Eng. Sc. in 1981 at Melbourne University. His work has been in the areas of finite element analysis, fracture mechanics and structural mechanics of composite and bonded repairs, and military aircraft accident investigations. He has also been involved with design studies of low radar cross-section battlefield surveillance R.P.V.'s. In 1985 he was seconded to the USAF at Eglin AFB for 18 months, to carry out vulnerability studies on composite structures. More recently he has been involved in a specific program on validation of bonded repairs to RAAF aicracft, and bonded repairs subject to acoustic fatigue.

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Dr. Chun H. Wang is currently a Principal Research Scientist and the Functional Head Damage Mechanics in the Airframes and Engines Division. He has made important contributions in fatigue and fracture mechanics, constitutive modelling, and advanced composite materials. Prior to joining DSTO in 1995, he has held various academic positions at the University of Sheffield, UK, the University of Sydney, and Deakin University, Australia. .

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Dr Galea graduated in 1980 with a Bachelor of Engineering (Mech) from the University of Queensland with first class honours and in 1983, he received a Masters of Engineering Science. He commenced employment with the Aeronautical Research Laboratory in 1983. In 1985 he commenced studies at the Institute of Sound and Vibration Research, University of Southampton, UK and received his Doctor of Philosophy from the University of Southampton in 1989. Dr Galea was appointed a Research Scientist in 1990 and Senior Research Scientist in 1992. Since 1990 Dr Galea has been working in the areas of composite structures, computational and experimental mechanics. He is currently managing the smart materials/structures for airframes task, at the Aeronautical and Maritime Research Laboratory. His

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Francis Rose graduated with a B.Sc (Hons) from Sydney University in 1971 and a PhD from Sheffield University, UK in 1975. He was appointed as a Research Scientist at the Aeronautical Research Laboratory in 1976 and is currently the Research Leader in Fracture Mechanics in the Airframes and Engines Division.
He has made important research contributions in fracture mechanics, non-destructive evaluation and applied mathematics. He is the regional Editor for the International Journal of Fracture and a member of the editorial board of Mechanics of Materials. He is also a Fellow of the Institute for Applied Mathematics and its Applications, UK and a Fellow of the Institution of Engineers, Australia.

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Mr. Sanderson has worked at AMRL since 1981. He has developed flight data reduction & analysis software for Mirage, F-111 & F/A-18 projects. Several of these programs have been implemented by NAE for part of their data reduction in the IFOSTP project. Since 1992, Mr. Sanderson has undertaken finite element analysis of composite and bonded structures for the F-111 and F/A-18 aircraft and provided training in finite element modelling and analysis.

Contents

1 Introduction

The repair of cracked aircraft structures subject to in-plane loads using bonded repairs has resulted in considerable aircraft life time extension and hence cost savings [1]. However the use of bonded patches to repair panels with acoustically induced cracks (acoustic fatigue) is only recent and presents some difficulties. Acoustic fatigue is a result of high frequency lateral vibration of an aircraft panel due to time varying pressure waves caused by engine and/or aerodynamic effects. For instance, acoustically induced cracks have been recorded in the lower external surface of the nacelle skin of the F/A-18 aircraft, as shown in Fig. 1. In this case the overall sound pressure levels of the order of 170 db have been recorded, [2] and [3]. Attempts to repair these cracks by applying standard methods of bonded repair [4] developed for in-plane loads were made, however the cracks continued to grow. The application of bonded repairs to acoustically-induced cracks requires analytical tools that take into account high frequency out-of-plane vibration. It has been found [5] that while the boron fibre composite bonded repairs did reduce the stress intensity, at the crack-tip, the high number of cycles lead to significant crack growth of up to $2mm$ per flight hour.

Since it is known that the amplitude of vibration is inversely proportional to the square root of the damping, then a combination of damping and stiffness may reduce the crack growth rate significantly. The application of highly damped repairs (Durability Patches) to acoustically damaged panels have been proposed by [6]. Furthermore [7] have applied a highly damped repair to the vertical fin of the F-15 aircraft, as part of a test program.

In this paper an analytical solution will be developed to allow the computation of stress intensity factors in highly damped repairs, which involves the use of crack-bridging theory [8]. A correction factor has been introduced to extend the crack bridging theory to finite width rectangular panels.
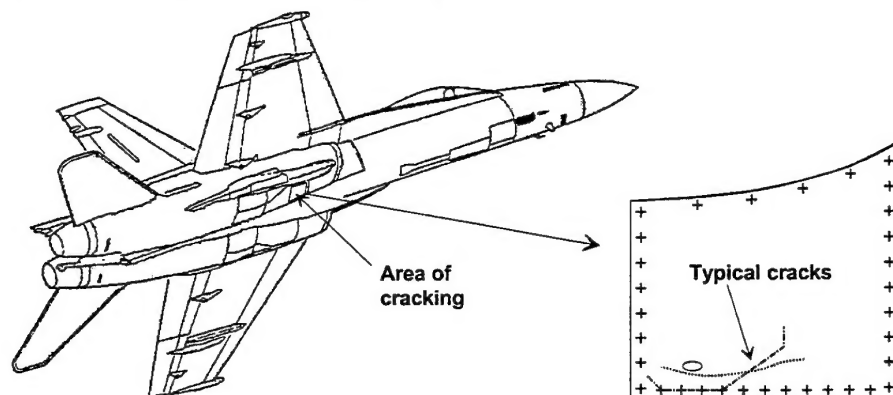

Figure 1: Location of the cracking in the lower nacelle inlet.

2 Analytical Approach

Acoustic fatigue is associated with the random loading of a structure over a wide frequency range. In the case considered here the loading is applied to a rectangular panel shown in Fig. 2, with three different sets of boundary conditions. The complete analytical solution of the dynamic response of the panel is very complex [9]. However in the case of acoustic fatigue a number of simplifying assumptions can be made as the emphasis here is to determine the crack growth driving force. Firstly the time varying pressure can be considered to be uniform and in phase over the panel. Secondly, since the first mode contributes to 98% of the panel response, [3], then the contribution of all other modes will be ignored. Also the variation of power spectral density (PSD) may be assumed to be constant with frequency near the first mode frequency. Based on these assumptions the root-mean-square of the stress response is given [10,11] as:

$$\sigma_{b,rms} = \sqrt{\frac{\pi f_0 S_I(f_0)}{4\delta}} \sigma_b \qquad (1)$$

where $\sigma_{b,rms}$ is the resultant root-mean-square stress at any point b in the plate (MPa)

σ_b is the stress at point b due to a static unit pressure (MPa/Pa)

$S_I(f_0)$ is the PSD of the excitation at f_0 (Pa²/Hz)

f_0 is the first resonant frequency (Hz) which depends on boundary conditions

δ is the viscous damping ratio for the overall structure

A similar expression for equation 1 [9] can be derived for displacements.

Since the stress intensity is proportional to the prospective stress (σ_{rms}) prevailing in the uncracked plate subjected to random acoustic pressure, then the following equation also holds:

$$K_{rms} = \sqrt{\frac{\pi f_0 S_I(f_0)}{4\delta}} K \qquad (2)$$

where K_{rms} is the root-mean-square (in time space) of the stress intensity factor and K is the stress intensity factor due to the application of a unit pressure on a cracked structure with or without repairs. For a repaired structure, K will be evaluated using crack-bridging theory. The problem is idealised as shown in Fig. 2a in which the plate is simply supported along two edges (cylindrical bending) subject to a random loading. The first resonant frequency is given by:

$$f_0 = \frac{\pi}{2}\sqrt{\frac{D}{\rho t_p L^4}}$$ (3)

where $D = \dfrac{E t_p^3}{12(1-\upsilon^2)}$,

E is Young's modulus,

t_p is the thickness of the plate,

ρ is the density ,

L and b are the length and width of the plate respectively.

For the case of a plate with all edges restrained, Fig. 2b and 2c, the first resonant frequency is given by:

$$f_0 = \frac{1}{2\pi} f(2b/L)\sqrt{\frac{D}{\rho t_p L^4}}$$ (4)

and $f(2b/L)$ is given in [12] for either or all simply supported or all clamped edges and is shown in Fig. 3.

(a) simply supported on 2 edges   (b) simply supported on all edges
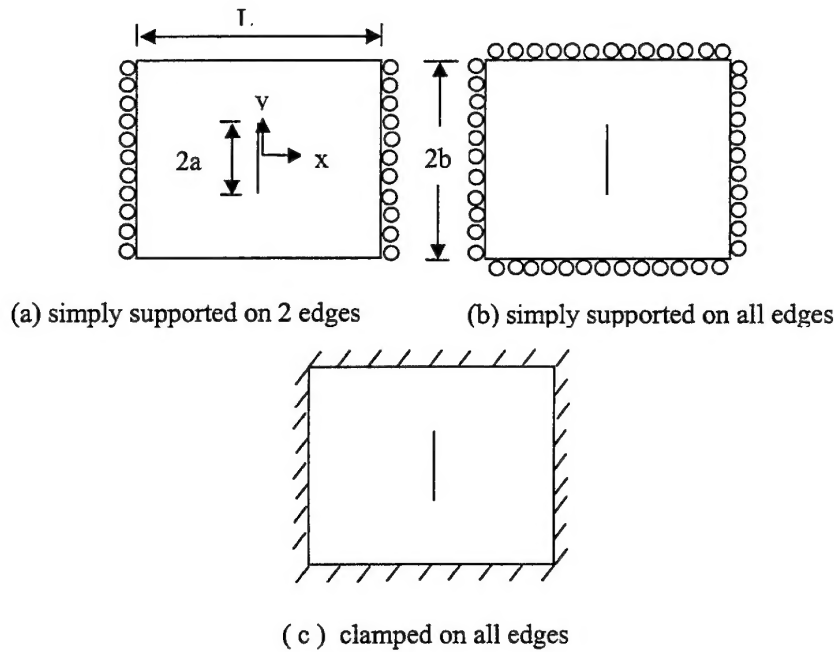
(c) clamped on all edges

Figure 2:   Edge support conditions for panels.
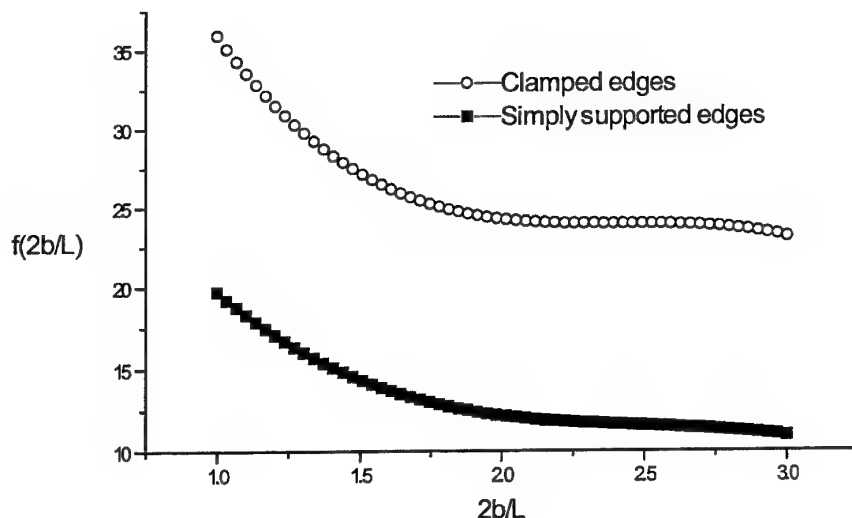
3

Figure 3: Frequency function for first mode of a rectangular plate, all edges simply supported or all edges clamped.

While in practise most acoustic fatigue cracking occurs at the edge of panels, for simplicity the crack has been assumed to be at the centre of the panel as shown in Fig. 2. For the case of simply supported edges, the bending moment that is experienced by the crack at the centre of the plate is higher than that at the edge. This location will also simplify the use of crack bridging theory. The size of the panel consided here is *b=126mm* and *L=195mm*.

3 Sound Pressure Levels

In this case, the sound pressure levels (SPL) measured on the external surface of F/A-18 inlet nacelle will be used, and are plotted in Fig. 4 with increasing frequency. The spectrum level, relative to the overall sound pressure level (OASPL), was derived from in-flight one third octave SPL measurements, [2]. This spectrum level is now used to calculate $S_I(f_0)$. The relationship between the spectrum SPL and the r.m.s. fluctuating pressure p, (units are Pa), is given by [13] as:

$$p_{rms} = 10^{(SPL/20-4.7)} \qquad (5)$$

and the power spectral density of acoustic pressure, i.e. PSD of the excitation, at any given frequency is given by:

$$S_I(f_o) = p_{rms}^2 = 10^{(SPL/10-9.4)}$$ (6)

The curve in Fig. 4 has been approximated by two straight lines defined by the three points listed in Table 1 and shown in Fig. 4. This approximate spectrum is used as the excitation PSD for the F.E. analysis.

Table 1: Input power spectral density.

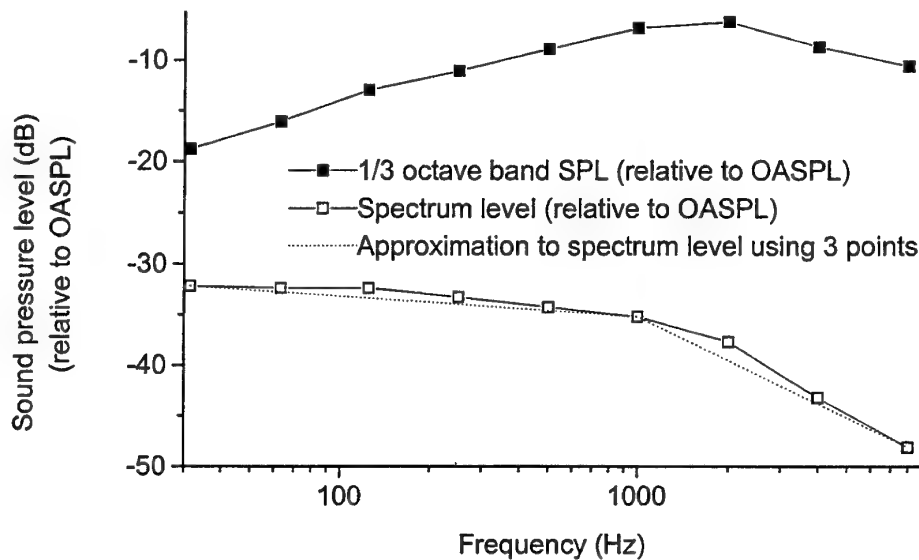| Frequency (Hz)	Three points to appromimate curve (dB)	Pressure spectrum level (dB)	$S_I(f_0)$ $(MPa)^2/Hz$
31.5	-32.2	140	4.0×10^{-8}
1000.	-35.2	137	2.005×10^{-8}
8000.	-48.1	124.1	1.028×10^{-9}

Figure 4: Spectrum and one-third octave band levels of sound pressure over the external nacelle inlet (where OASPL=170 db) [2].

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4 Crack Bridging Theory

The most common type of repairs to be applied to aircraft skins are one sided as illustrated in Fig. 5 since, in general, access is only available to the outside of the skin. Analytical solutions for one sided repairs are available for both in-plane [14] and out-of-plane loading [8]. A significant feature of bonded repairs is that the variation of stress intensity $K(z)$ with crack length does not increase indefinitely with crack length. Instead, the stress intensity approaches an asymptotic value. The thickness of the adhesive is included in the analysis to represent the damping material. Crack bridging analysis for the case shown in Fig. 5b gives the solution, shown in Fig. 6, as:

$$K(z) = \sigma_b . f_m + \sigma_b \frac{2z}{t_p} f_b \qquad (7)$$

where f_m and f_b denote the membrane and bending components of the normalised stress intensity factor induced by a uniform bending stress, which varies linearly through the plate thickness [15]. σ_b is the maximum bending stress in absence of a repair and crack. Both f_m and f_b depend on:

E_P, Young's modulus for the skin,

E_R, Young's modulus for the reinforcement,

t_p, the thickness of the skin,

t_a, the thickness of the adhesive or damping material,

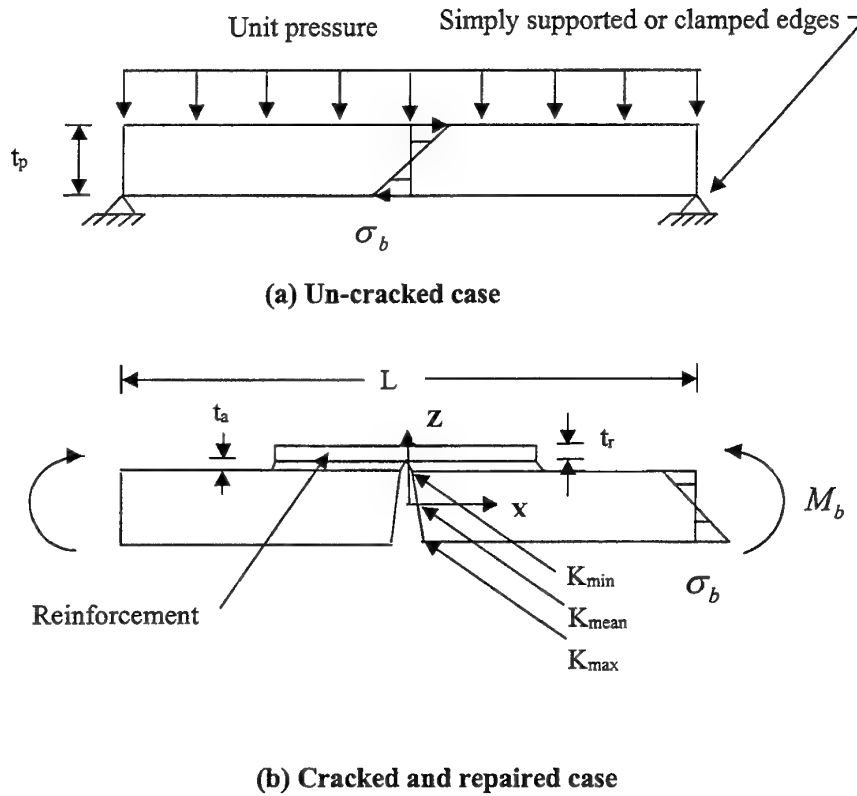t_r, the thickness of the reinforcement,

L, the length of the beam.

(a) Un-cracked case

(b) Cracked and repaired case

Figure 5: *One sided repair consisting of a reinforced beam subject to one unit of lateral pressure.*

Table 2: Material properties.

Material	Young's modulus (GPa)	Poisson's ratio	Density (Mg/mm³)
Aluminium	71.	.33	2.77×10^{-9}
Adhesive [16]	2.3	.35	1.2×10^{-9}
Boron [17]	207.	.21	2.0×10^{-9}

The crack–bridging model involves a reinforced plate with a central crack, as shown in Fig. 5b. It is worth noting that, for a unit pressure, σ_b needs to be calculated using the appropriate boundary conditions, e.g. simply supported or clamped. The crack–bridging model plots of $K(z)/\sigma_b$ in Fig. 6 show the maximum, minimum and mean values of stress intensity, (K_{max}, K_{mean}, and K_{min}). In this case, the material properties and dimensions are given in Table 2. The stress σ_b is the bending stress at the centre of the

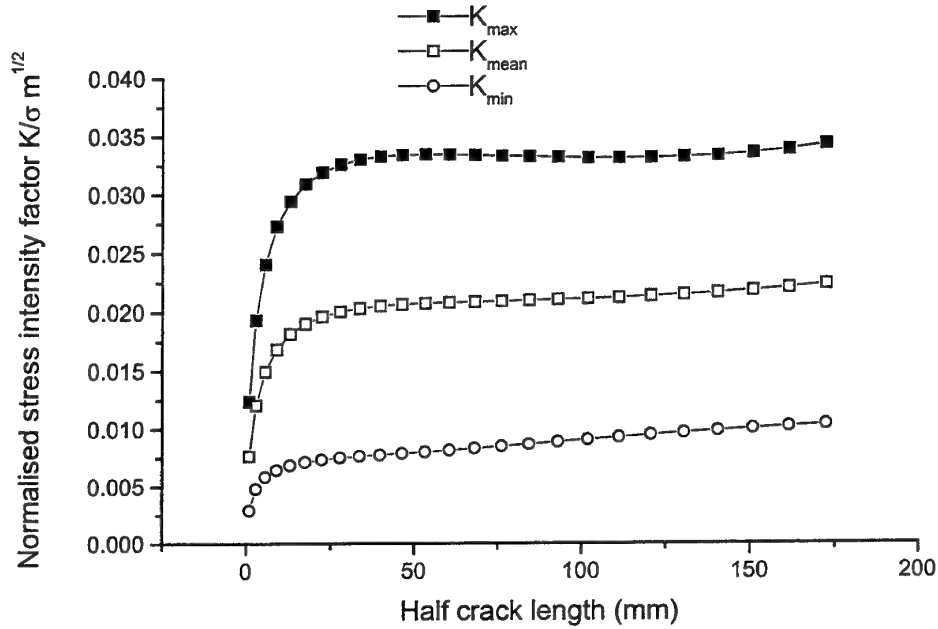beam in the absence of a repair and a crack. Note that Fig. 6 shows the expected asymptotic behaviour.

Figure 6: Results from crack bridging model, [Wang and Rose] for constant lateral static loading of a beam.

Firstly the mean stress intensity factor is given by:

$$K_{mean} = \frac{K_{max} + K_{min}}{2}$$ (8)

and the bending stress intensity factor is given by:

$$K_b = \frac{K_{max} - K_{min}}{2}$$ (9)

These equations will be considered futher on.

For a simply supported plate, Fig. 2a, subject to a uniform pressure of unit magnitude, the maximum bending stress is given by:

$$\sigma_b = \frac{3L^2}{4t_p^2} \tag{10}$$

In the context of repairs σ_b corresponds to the nominal far-field bending stress. Hence using equations(1 and 10) the root-mean-square bending stress of a simply supported plate along two edges due to the time varying random pressure with PSD, $S_I(f)$ is given by:

$$\sigma_{b,rms} = \sqrt{\frac{\pi f_0 S_I(f_0)}{4\delta}} \frac{3L^2}{4t_p^2} \tag{11}$$

5 Finite Element Approach

The random response analysis capability of the NASTRAN program [18] has been used to validate the analytical results presented here. The system response to random excitation is evaluated via the frequency response technique viz.,

$$S_J(\omega) = |H(\omega)|^2 S_I(\omega) \tag{12}$$

where in this case $H(\omega)$ is the frequency response of a displacement u_J due to an excitation force (pressure) p_I. S_J and S_I are the PSD of the response (displacement) and of the excitation (pressure), respectively. This analysis also allows the statistical properties of the system to be evaluated. Random vibrations considered here involve all frequencies at any one instant in time. After calculating the PSD, the root-mean-square (r.m.s.) of the response can be determined by computing the square root of the PSD area:

$$j_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\infty S_j(\omega)\, d\omega} \tag{13}$$

where the limits of the integral will be truncated as shown in Fig. 4. No structural response is expected below 30Hz and the excitation levels above 8000Hz are minimal.

A similar application of finite element techniques to undertake a PSD analysis to acoustic fatigue problems has been reviewed by [13].

5.1 Computation of stress intensity factors using F.E.

In the F.E. model the thickness of the plate is modelled using three layers of 20 noded brick elements in order to account for the bending behaviour of the skin. The skin thickness is approximately 1 mm, hence the condition of plane stress is assumed. The computation of the stress intensity factor is determined directly from the crack opening displacements near the crack tip using displacements from the PSD analysis. The r.m.s. crack tip stress intensity factors for mode I are derived from the standard asymptotic relation [20]:

$$K_{rms} = \frac{EU_{rms}}{4}\sqrt{\frac{2\pi}{l}}$$

(14)

where K_{rms} is the root-mean-square stress intensity factor evaluated at the desired through thickness location

E is Young's modulus

U_{rms} is the root-mean-square of the crack opening displacement at distance l from the crack tip (perpendicular to the plane of the crack) and at the desired through thickness location.

The F.E. model is comprised entirely of 20 noded brick elements. For the plate three elements are used through the thickness while two elements are used through the thickness for the adhesive and patch. The size of the crack tip element ($l = 0.15625mm$) has been selected such that $l << 1/k$, where $k = (\sigma_b / K_{max})^2 \approx 1.0mm$, where values of K / σ_b are taken from Fig. 6.

6 Analysis of Results

6.1 Plate with two edges simply supported

6.1.1 Plate with no crack and no repair

Consider the configuration shown in Fig. 2a in which the plate is simply supported at two edges but does not contain a crack or a repair. The material properties of this plate are shown in Table 2 while the dimensions are shown in Table 3. As an approximation the stress σ_b at $x=0$ of the plate subject to unit pressure is given by equation(10). The frequency of the first mode (69 Hz) is computed using equation(3). The corresponding PSD of the excitation at f_0, $S_I(f_0)$ is interpolated from the spectrum listed in Table 1,

and is shown in Table 4. Using equation(1) the rms stress (σ_{rms}) is evaluated, with a viscous damping ratio of 0.108 at the centre and on the surface of the plate and is given in Table 5. Also listed in Table 5 is the F.E. result which in this case, shows a disagreement of 4.4% compared to the analytical bending stress results.

Table 3: Dimensions of plate shown in Fig.2.

L (mm)	2b (mm)	t_p (mm)	t_a (mm)	t_r (mm)
195.	252	1.143	0.254	0.635

Table 4: Dynamic properties of cracked and repaired plates

No. edges restrained	f_o (Hz)	S_i (f_o) (MPa)²/Hz	δ
2	69 (equation 3)	3.917×10^{-8}	.108
4	118 (equation 4)	3.822×10^{-8}	.108

Table 5: Stress results for un-cracked, un-repaired plate.

Theoretical (MPa)	F.E. result (MPa)	Error %
96.8	92.7	4.4

6.1.2 Plate with crack and repair

In this case the properties of the repaired plate are also contained within Tables 2 and 3. The patch is assumed to cover the full width of the plate. The following equation, derived from equations(2 and 8), has been used to calculate the root-mean-stresses where values of K_{max} / σ_b, K_{min} / σ_b are given in Fig. 6:

$$K_{rms} = \sqrt{\frac{\pi f_0 S_I(f_0)}{4\delta}} \frac{3L^2}{4t_p^2} \left(\frac{K}{\sigma_b} \right) \tag{15}$$

where the frequency is given by equation(3).

Using equations(8 and 9) the ratio of $(K_{mean})_{rms} / (K_b)_{rms}$ has been computed for both F.E. and analytical analysis and is shown in Fig. 7. The best agreement occurs at a half crack-length of approximately 20 mm and corresponds to an error of 6.7%. The largest error occurs at the smallest and largest crack length. An average error between these results is 9%. The characteristic of the F.E. results is the constant value of

$(K_{mean})_{rms}/(K_b)_{rms}$ as the crack length increases while in the case of theory the ratio has yet to stabilise. Also the closed form solution results are always higher than the F.E. results.
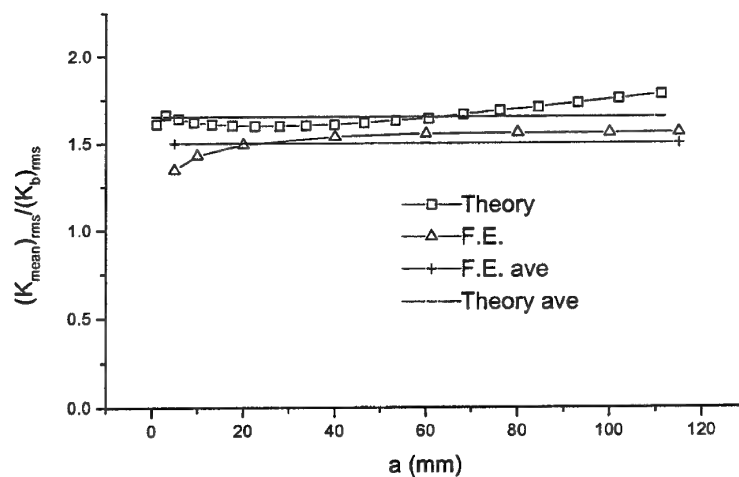

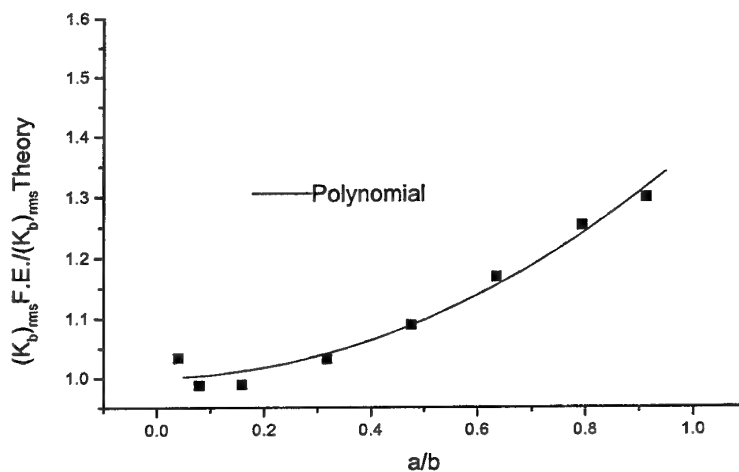Figure: 7: Comparison of $(K_{mean})_{rms}/(K_b)_{rms}$ for theory and F.E.

Figure 8: Plot of $(K_b)_{rms}$ F.E. $/(K_b)_{rms}$ Theory, polynomial fit: $1.+0.011(a/b)+0.366(a/b)^2$

This error may be due to an edge effect as $a/b{\to}1$. In order to make a correction factor $(K_b)_{rms}^{F.E.}/(K_b)_{rms}^{Theory}$ has been plotted against a/b, as shown in Fig. 8, and fitted with a second order polynomial such that the results are assumed to be correct at small crack-

lengths. It is noted that the ratio $(K_{mean})_{rms}^{F.E.}/(K_{mean})_{rms}^{Theory}$ should always be one. This polynomial is defined below where the values of a and b are defined in Fig. 2:

$$f = (K_b)_{rms}^{F.E.}/(K_b)_{rms}^{Theory} = 1.0+0.011(a/b)+0.366(a/b)^2 \qquad (16)$$

As a result the following correction is applied:

$$(K_b)_{rms} = f(K_b)_{rms}^{Theory} \qquad (17)$$

$$(K_{max})_{rms} = (K_{mean})_{rms}^{Theory} + (K_b)_{rms} \qquad (18)$$

$$(K_{min})_{rms} = (K_{mean})_{rms}^{Theory} - (K_b)_{rms} \qquad (19)$$

where $(K_{mean})_{rms}^{F.E.} = (K_{mean})_{rms}^{Theory}$

A comparison with the corrected theory and F.E. results for $(K_{max})_{rms}$, $(K_{mean})_{rms}$ and $(K_{min})_{rms}$ are shown in Fig. 9. The corrected values of $(K_{max})_{rms}$, $(K_{mean})_{rms}$ and $(K_{min})_{rms}$ for crack bridging theory are all conservative.
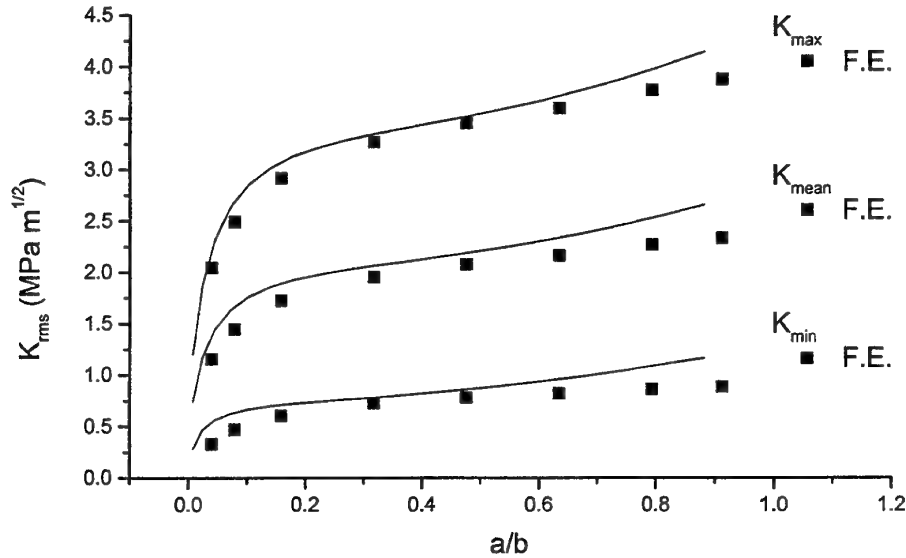

Figure 9: Comparison of analytical and F.E. results for two edges simply supported.

In the analytical solution it is assumed that the frequency does not change with crack length. The F.E. values of first resonant frequency with crack length ratio are shown in Table 6, and indicate that only a small change of frequency occurs. (i.e., frequency change of 1% compared to the original first resonant frequency for crack length ratio of a/b=.92).

Table 6 Frequency of repaired plate versus crack length ratio for two edges simply supported.

Crack length ratio a/b	Frequency Hz
.0397	72.43
.0794	72.41
.1587	72.33
.3175	72.13
.4762	71.90
.6349	71.64
7937	71.36
.9127	71.14

6.2 Plates simply supported on all edges

In general aircraft skin panels are usually connected to other plates or stiffeners at their boundaries, consequently the acoustic response lies somewhere between the limiting cases of simply-supported edges or fully-fixed edges. Simple plate theory can be used to determine the bending moment distribution in the plate resulting from a uniform pressure distribution. In the case of four simply supported edges the bending distribution relative to the crack location is shown in Fig. 10, where the plate is of dimensions 2b by L. The average bending moment can be determined from [20] as:

$$M_{ave.} = \beta L^2 \tag{20}$$

Note that β is a function of (a/b and b/L) and is shown in Fig. 11.

Since: $\sigma_b = 6M/t_p^2$ then from equation(20):

$$\sigma_b = 6\beta L^2 / t_p^2 \tag{21}$$

The stress intensity factor is approximately equal to that corresponding to a uniform bending moment with magnitude being equal to the average over the crack length.

Hence from equation(2):

$$K_{rms} = \sqrt{\frac{\pi f_0 S_I(f_0)}{4\delta}} \frac{6\beta L^2}{t_p^2}\left(\frac{K}{\sigma_b}\right) \qquad (22)$$

where $K(z)$ is given by equation(7). In this case the frequency f_0 is given by equation(4) and $S_I(f_0)$ is interpolated from Table 1, and is shown in Table 5.
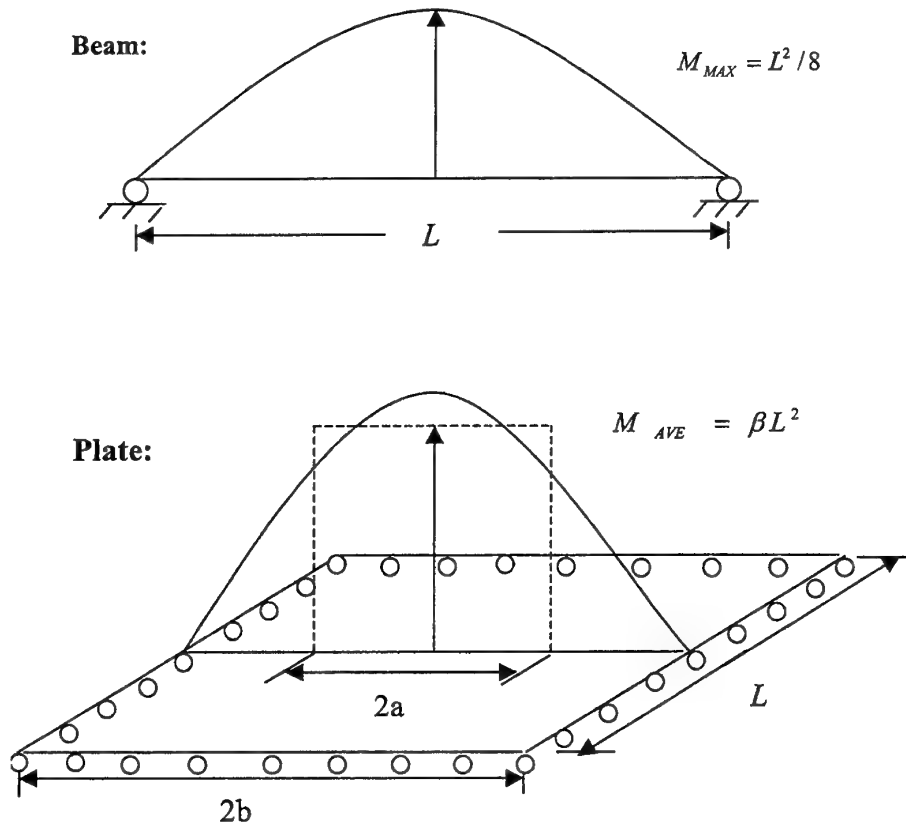
Beam:

$$M_{MAX} = L^2/8$$

$$L$$

Plate:

$$M_{AVE} = \beta L^2$$

2a

L

2b

Figure 10   Comparison of bending moments from beam and plate with simply supported edges.

The closed form result of $(K_{max})_{rms}$, shown in Fig. 12, gives conservative agreement over the complete crack length range. It is evident in the problem considered that for an aspect ratio of $2b/L \approx 1$, the edges (whose dimension is L as shown in Fig. 10) have a strong restraining influence on the bending developed in the repair, i.e. the F.E. results show lower values than those from the closed form solution. For large aspect ratio's $(2b/L >> 1)$ it would be expected that closer agreement would occur. A closed form

solution in which the moment at the crack tip is considered and is also shown in Fig. 12, but gives unconservative results for large crack lengths. Note that no edge correction factor has been used.
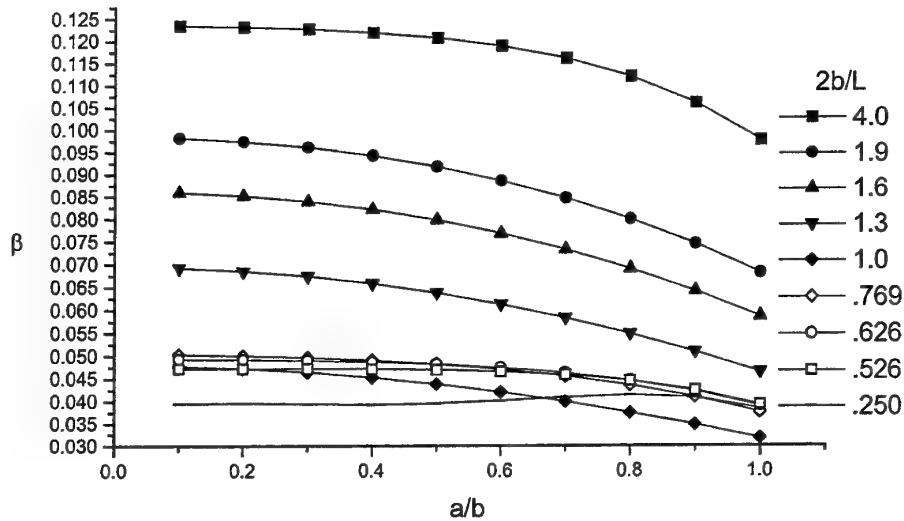

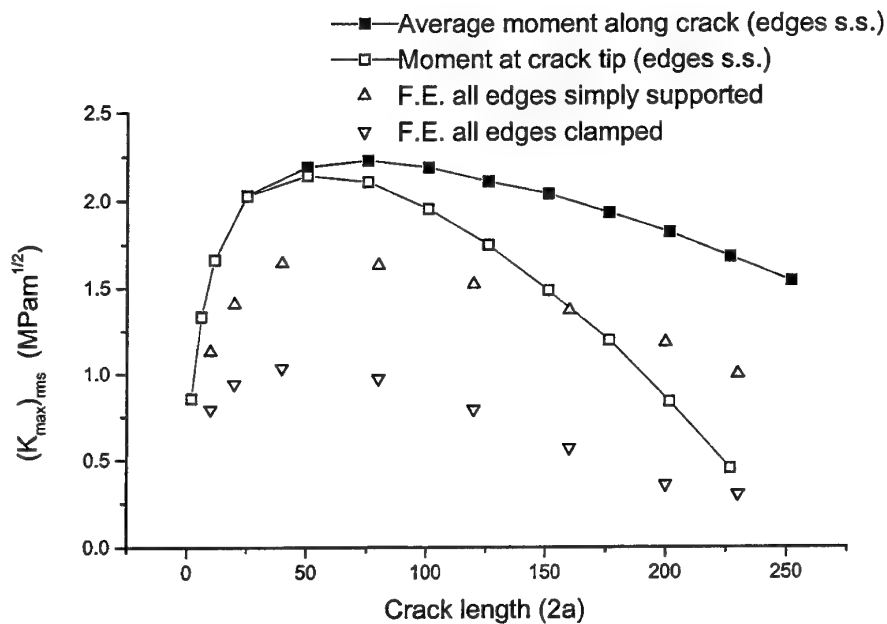*Figure 11: Parameter β derived from plate bending theory for simply supported edges.*

Figure 12: Comparision of $(K_{max})_{rms}$ calculated by crack bridging theory and F.E

6.3 Plates clamped on all edges

The F.E. results for built in edges are also shown in Fig. 12 however an appropriate correction factor has not been applied to crack bridging theory. The solution is also applicable for cracks along the boundary, which generally occur in practice, however the result will be conservative since the bending moment will be lower at the boundary than that at the centre of the plate.

6.4 Effect of damping

For the case of a plate with all edges simply supported the closed form solution has been used to consider the influence of damping. Results shown in Fig. 13, are a plot of $(K_{max})_{rms}$ with viscous damping coefficient, and demonstrates the possible reduction in stress intensity factor with increased damping. The low values of damping would correspond to a metallic riveted aircraft skin [2], and the high values to a panel treated with high damping. It is evident that for a highly damped repair the stress intensity factor may be further reduced by a factor of 2.5.
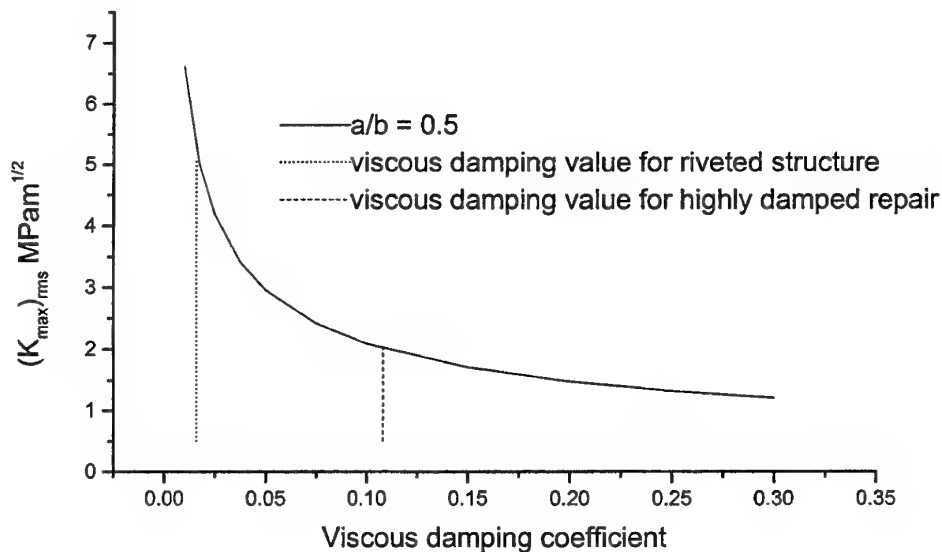

Figure 13: Variation of Stress intensity factor with increasing viscous damping coefficient. (Note the reduction of stress intensity factor for highly damped repair in comparison to metallic riveted structure).

6.5 Power spectral density of the response (PSD)

The PSD of the response using F.E. is shown in Fig. 14 for the case of a highly damped repaired plate simply supported on all edges. Since the response is given by equation(12), then a running integral of the PSD of the response will give the

contribution of each mode. A COD method is used to determine the stress intensity using displacements at the crack tip. It can be seen that the first mode, at a frequency of 118 Hz, dominates the response of the structure compared to the higher modes. A running integral of this curve shows that the first mode contributes to 95% of the response. While there are extreme cases in which this does not occur [13], in general however, the first mode does contribute to most of the response, and hence this assumption made to derive the analytical solutions for K_{max} is accurate to the degree shown in this work.
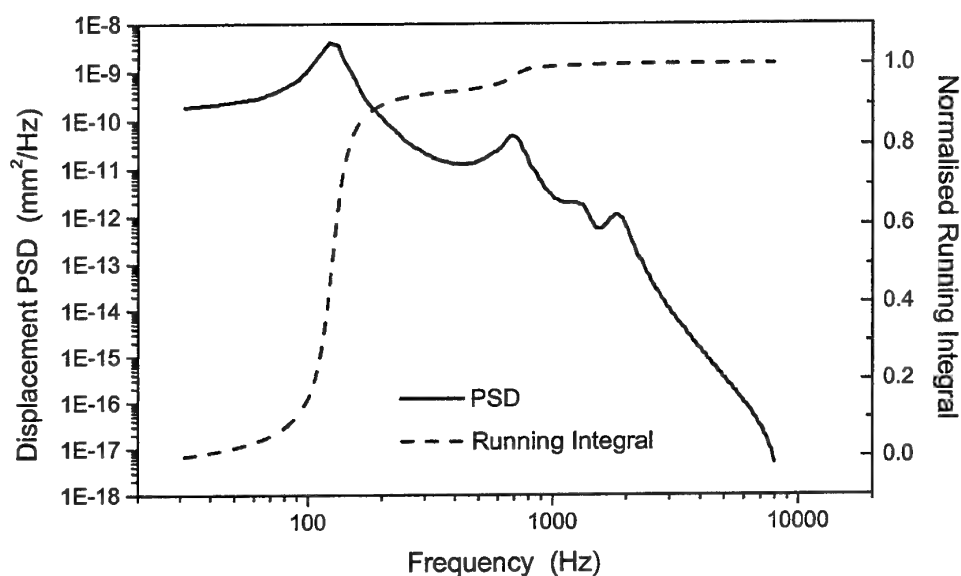

Figure 14:   The PSD of the displacement response for a plate, with a highly damped repair, simply supported on all edges.

7 Conclusions

In this paper an analytical solution has been developed using a simplified dynamic analysis together with crack bridging theory to give a useful upper bound for idealised configurations. It has been shown that the analysis for crack bridging can predict the behaviour for plates in cylindrical bending and rectangular panels restrained on all edges. Furthermore the solution is also applicable for plates containing cracks on the boundary, and this solution will be conservative. As a result this theory together with

appropriate crack growth data, and damping data will aid in the design of highly damped repairs of acoustically-induced cracked panels.

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19. ABSTRACT

The application of bonded repairs to structures subject to acoustic fatigue is investigated, with a view to developing simple solutions suitable for engineering applications. Acoustic fatigue is caused by a random response of a structure due to time varying pressure waves from engine and/or aerodynamic sources. For vibration problems the use of damping in repairs will reduce the amplitude of vibration and hence the stress intensity factor in the cracked structure. To assess the viability of the composite bonded repairs subject to acoustic fatigue new analytical tools are needed. In this paper the use of a simplified dynamic analysis combined with crack-bridging theory allows the influence of damping to be assessed. This theory is adapted to panels with various edge support conditions with good results. Validation of this work is carried out using a Finite Element (F.E.) analysis in which three-dimensional brick elements are used to model the plate, adhesive and boron patch, and use is made of the power spectral density capability of the NASTRAN program.